Linewidth narrowing due to Level attraction in coupled metamaterial

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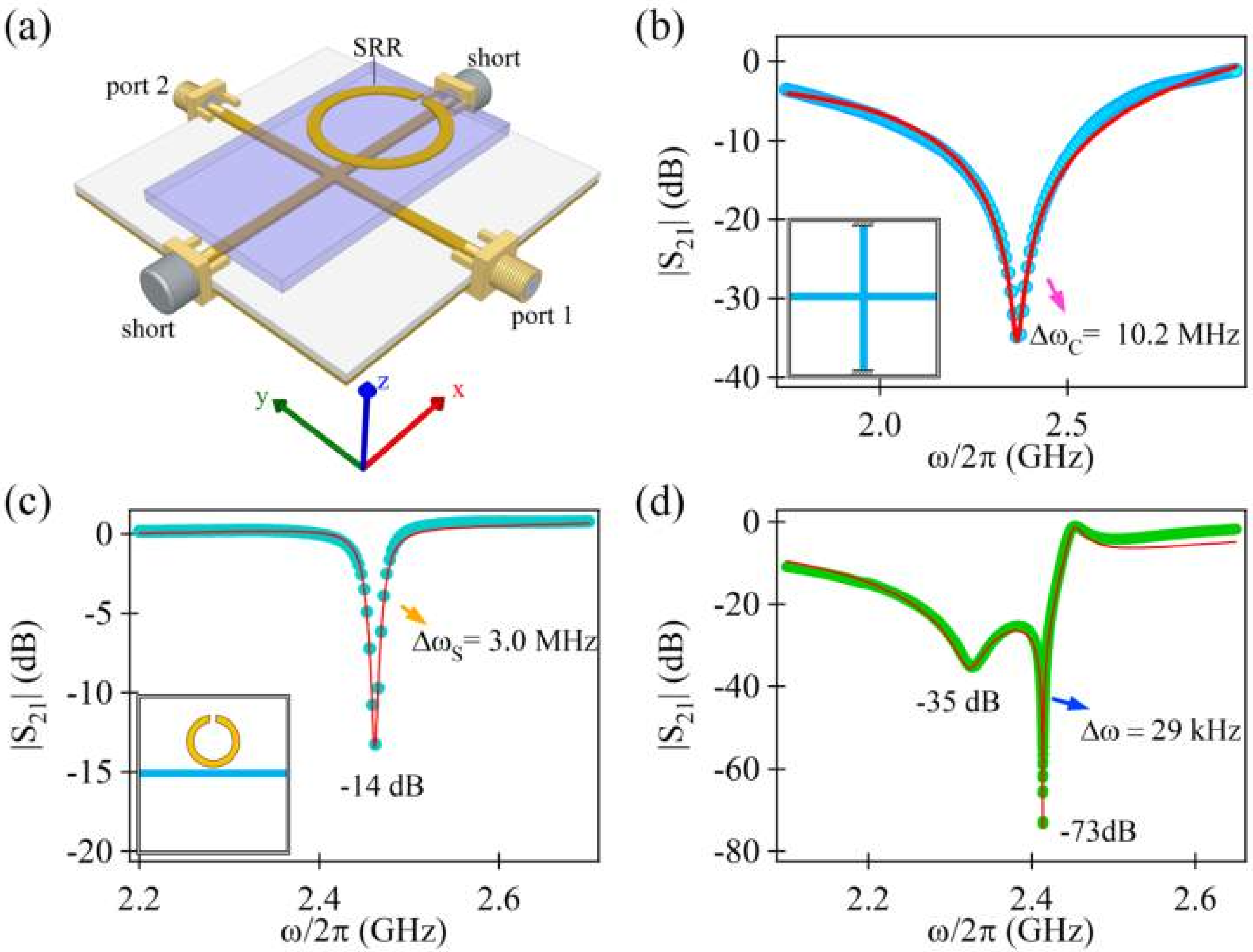


FIG. 1. (a) Schematic drawing of experiment setup with a microstrip cross junction cavity and a split-ring resonator with a outer radius (r = 9.5 μm). The arms in x direction have been shorted to ground. The node of the ring was placed right above the center of the cross and the gap was aligned with one shorted arm of the cross. (b) The transmission (S_{21} parameter) of the empty cross cavity with measurement (markers) and calculation (red curve). (c) The transmission of the empty cavity with measurement (light blue marks) and calculation (red curve). (d) The S_{21} of the coupled system shown in (a) which shows a sharp absorption resonance, the fitted curve calculated using Eqn.1.

(a)(d) will be modified

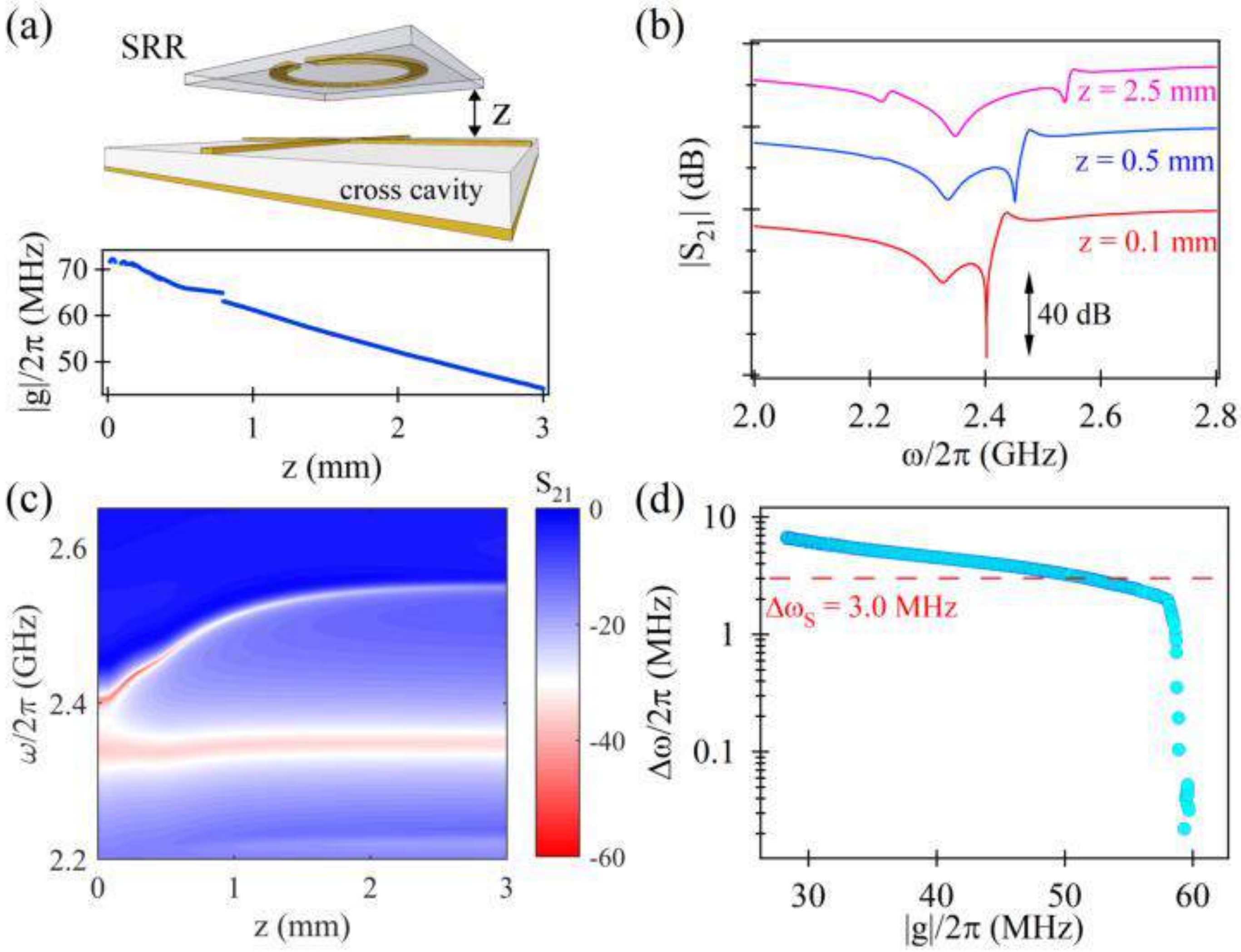


FIG. 2. (a) The experiment setup of determining the distance influenced coupling (Top) and the total coupling strength as a function of z (bottom). coupling strength obtained through fitting Eqn.1. (b) Typical spectra tuning the separation distance in z axis, a sharp resonance can be observed when the two structures are close to each other. (c) The spectra mapping of S_{21} as a function of the distance and driven frequency. As the coupling strength increase (decrease of the z value), the two modes attract each other. (d) The linewidth as a function of total coupling strength. As the coupling strength increasing, the linewidth drops two order of magnitude.

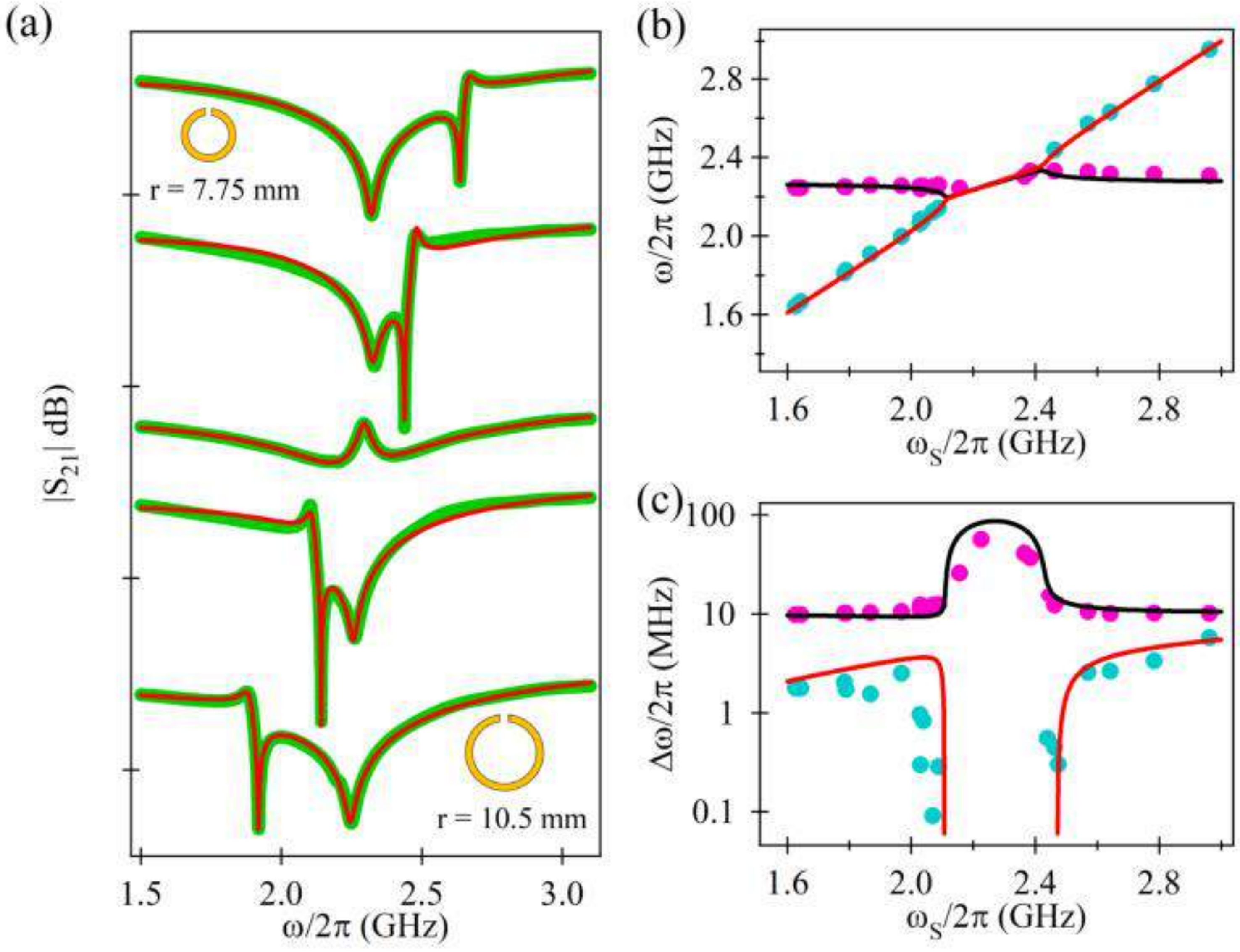


FIG. 3. Level attraction in passive metamaterials. (a) The waterfall plot of the level attraction with measurement (green markers) and fitting (red curves). (b) The dispersion using various SRRs coupled with cross cavity which shows an attractive behaviour. (c) The linewidth evolution of the coupled system with experiment (markers) and calculation (curves) where the linewidth has narrowed dramatically.

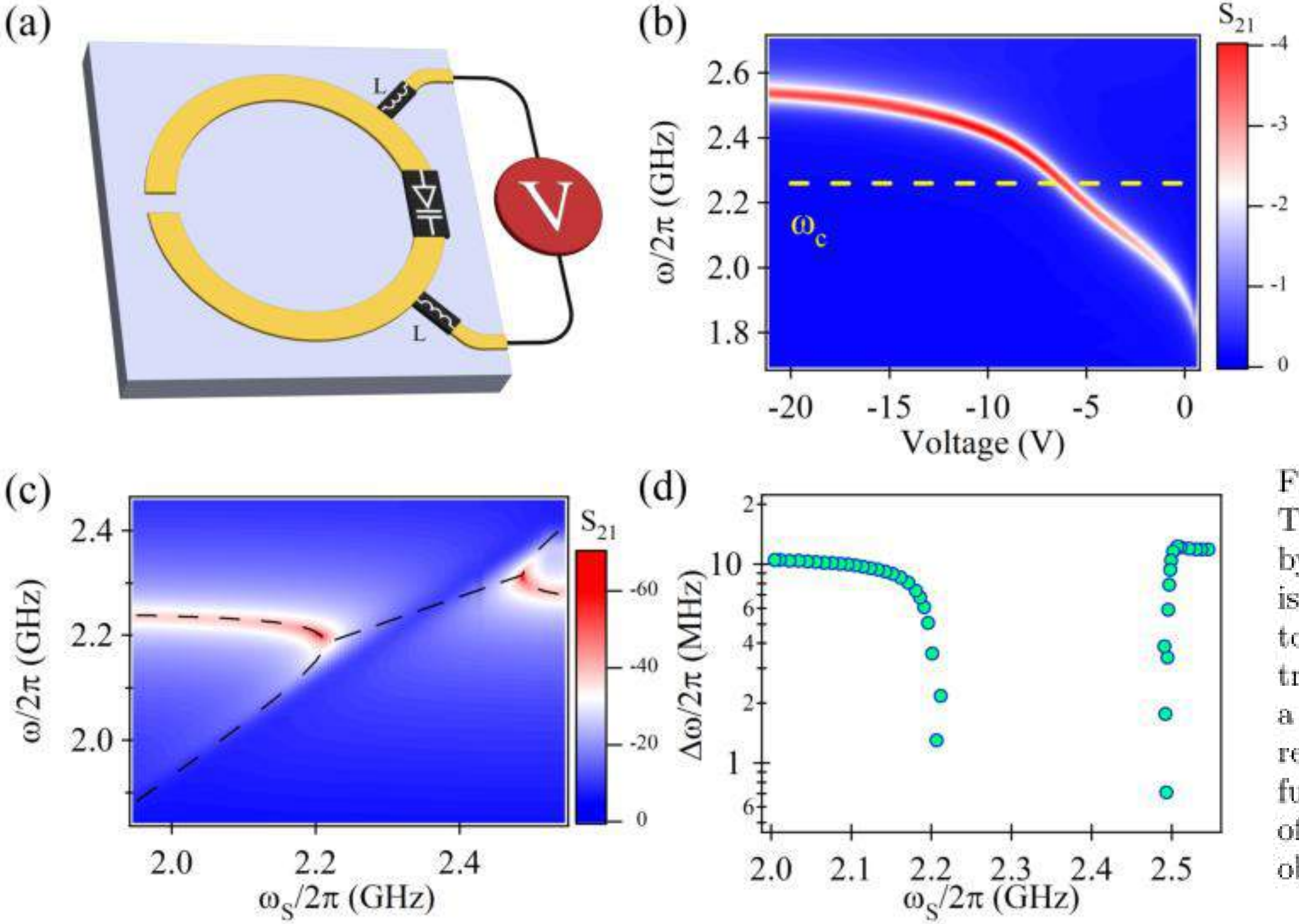


FIG. 4. Level attraction in active tunable metamaterials. (a) The design of the active tunable split-ring resonator controlled by a voltage source (in red). The black component in center is the varactor diode and the other components are inductors. (b) The dispersion of the varactor loaded SRR using a transmission line. (c) The dispersion of the coupled system as a function of resonant frequency of the SRR with calculated results shown in the dashed line. (this will be reproduced in future versions) (d) The evolution of linewidth as a function of frequency of SRRs, the linewidth narrowing can be clearly observed near the cavity resonance.

Lagrangian model (1)

Generalized coordinates:

The charge accumulated on a resonator $Q, \dot{Q}$

Kinetic energy stored in inductor: $T = \left(L\dot{Q}^2\right)/2$

Electrostatic energy stored in capacitor: $V = Q^2/2C$

For a single resonator, we have:

$$\omega = \sqrt{LC}$$

$$\mathcal{L} = T - V = \frac{L\dot{Q}^2}{2} - \frac{Q^2}{2C}$$

$$\mathcal{L} = \frac{L}{2}\left(\dot{Q}^2 - \omega^2 Q^2\right)$$

Consider the coupling between two resonators, the Lagrangian is

$$\mathcal{L} = \frac{L_1}{2}\left(\dot{Q}_1^2 - \omega_1^2 Q_1^2\right) + \frac{L_2}{2}\left(\dot{Q}_2^2 - \omega^2 Q_2^2\right) + M_H \dot{Q}_1 \dot{Q}_2 + M_E \omega_1 \omega_2 Q_1 Q_2$$

Mutual inductance   Mutual capacitance

Rayleigh's Dissipation function:

$$\mathcal{F} = \frac{\gamma_1}{2}\dot{Q}_1^2 + \frac{\gamma_2}{2}\dot{Q}_2^2 + M_R \dot{Q}_1 \dot{Q}_2$$

Mutual resistance

Lagrangian model (2)

Generalized Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Q}_i}\right) - \frac{\partial \mathcal{L}}{\partial Q_i} + \frac{\partial \mathcal{F}}{\partial \dot{Q}_i} = 0\,, \qquad i = 1,2$$

$$\ddot{Q}_1 + \gamma_1 + \omega_1^2 Q_1 + \frac{M_H}{L_1}\ddot{Q}_2 + \frac{M_E}{L_1}\omega_1\omega_2 Q_2 + \frac{M_R}{L_1}\dot{Q}_2$$

$$\ddot{Q}_2 + \gamma_2 + \omega_2^2 Q_2 + \frac{M_H}{L_2}\ddot{Q}_1 + \frac{M_E}{L_2}\omega_1\omega_2 Q_1 + \frac{M_R}{L_2}\dot{Q}_1$$

$$Q_1 = Ae^{i\omega t}$$

$$Q_2 = Be^{i\omega t}$$

$$\kappa_E = \frac{M_H}{L_1}\ddot{Q}_2$$

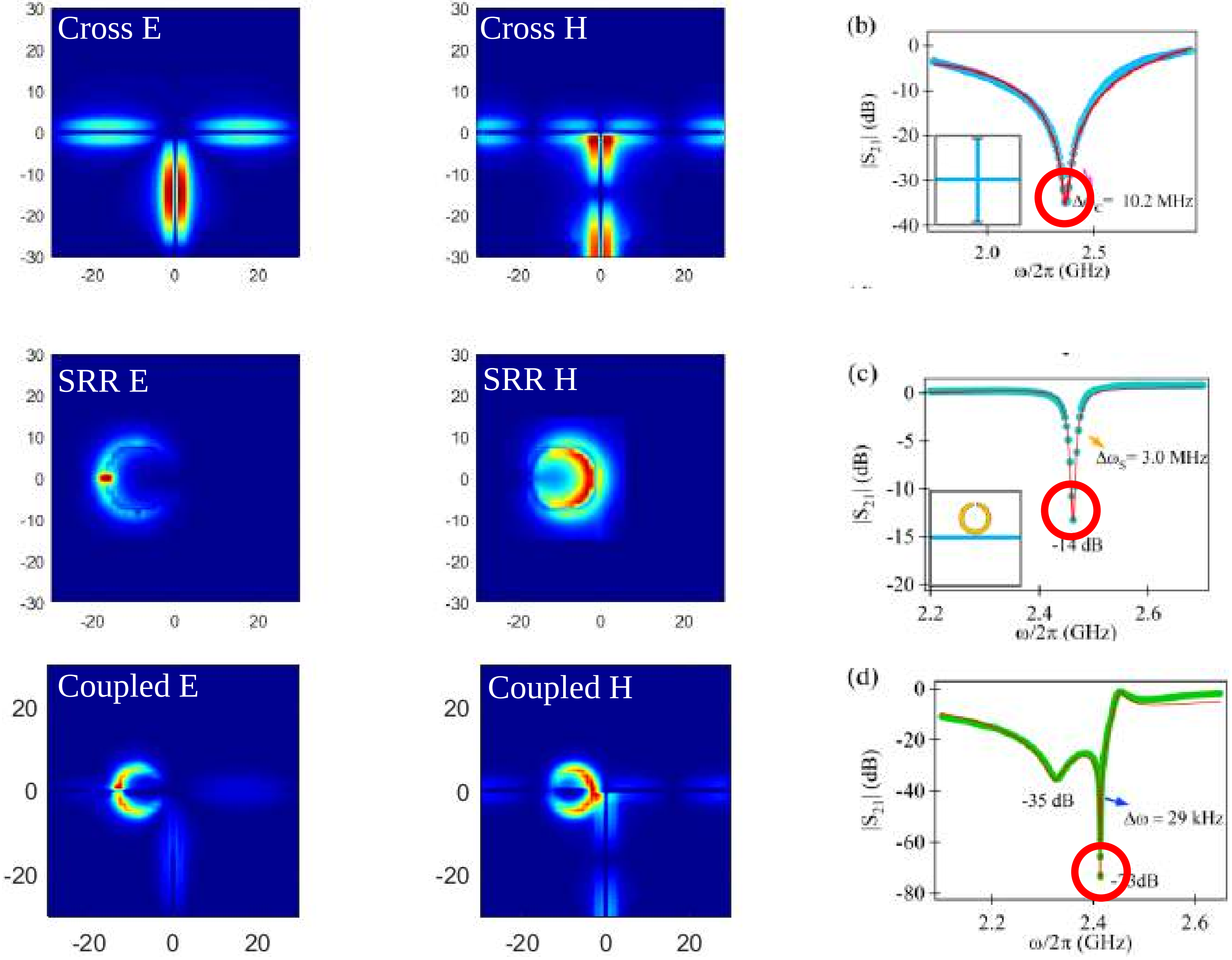
$$\kappa_H = \frac{M_E}{L_1}\omega_1\omega_2 Q_2$$

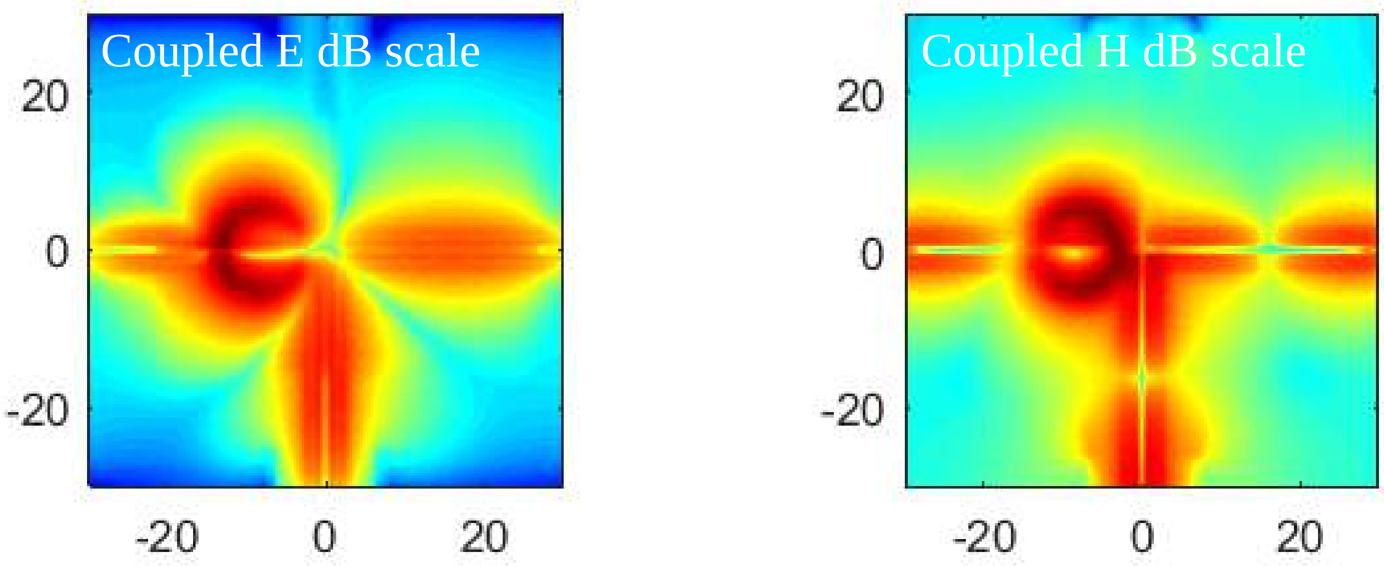
$$\kappa_R = \frac{M_R}{L_2}\dot{Q}_1$$

$$\begin{pmatrix} \omega_1^2 - \omega^2 + i\omega\gamma_1 & \kappa_E - \kappa_H + i\kappa_R \\ \kappa_E - \kappa_H + i\kappa_R & \omega_2^2 - \omega^2 + i\omega\gamma_2 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$g = J + i\Gamma$$

$$J = \kappa_E - \kappa_H,\ \Gamma = \kappa_R$$


Cross E
Cross H
(b)
|S21| (dB)
Δωc= 10.2 MHz
ω/2π (GHz)
SRR E
SRR H
(c)
Δωs= 3.0 MHz
-14 dB
Coupled E
Coupled H
(d)
-35 dB
Δω = 29 kHz
-73dB


Coupled E dB scale
20
0
-20
-20
0
20
Coupled H dB scale
20
0
-20
-20
0
20